2003 CHAPTER COMPETITION

SPRINT ROUND QUESTIONS

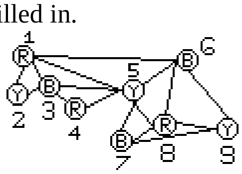
1. Raquel has \$3.80 in nickels and dimes. She has 48 nickels or \$2.40. This leaves \$1.40 to be represented in dimes. Therefore, Raquel has 14 dimes. 14 **Ans.**

2. A number is divisible by 4 if its last two digits are divisible by 4. (Consider that all multiples of 100 are divisible by 4. Thus, continually adding 4 to a multiple of 100 will always have multiples of 4 ending in the same digits., i.e., 104, 108, ... 196, 200, 204, 208, ... 296, 300, 304, 308, ... and so on.)
3544 is divisible by 4 because 44 is divisible by 4.
3564 is divisible by 4 because 64 is divisible by 4.
3572 is divisible by 4 because 72 is divisible by 4.
3576 is divisible by 4 because 76 is divisible by 4.
But 3554 is not divisible by 4 because 54 is not divisible by 4.
$5 \times 4 = 20$ **Ans.**

3. A game at Turner Field costs, on the average, \$66.56. Therefore, the couple can plan on spending \$133.12 for the two games at Turner Field. A game at Dodger Stadium costs, on the average, \$50.88.
\$133.12 + \$50.88 = \$184 **Ans.**

4. $1000 \div 13 = 76$ R 12
Since the remainder is 12,
1000 - 12 = 988 **Ans.**

5. The following figure shows the entire form filled in.



Circles 1 and 2 are filled in as R and Y, respectively. This means that Circle 3 must be B.
Circle 1 is R and Circle 3 is B. This means that Circle 5 must be Y.
Circle 3 is B and Circle 5 is Y This means that Circle 4 must be R.
Circle 1 is R and Circle 5 is Y. This means that Circle 6 must be B.
Circle 5 is Y and Circle 6 is B. This means that Circle 8 must be R.
Circle 5 is Y and Circle 8 is R. This means that Circle 7 must be B.
Circle 6 is B and Circle 8 is R. This means that Circle 9 must be Y. **Ans.**

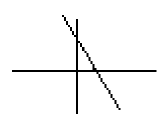
6. There are 25 cards with the numbers 1 through 25 on them. The multiples of 2 are 2, 4, 6, ... 20, 22, and 24. There are 12 or them. The multiples of 5 are 5, 10, 15, 20 and 25. Of course, 2 of them, 10, and 20, are already counted. Thus, there are 12 + 5 - 2 = 15 cards that have multiples of 2 or 5.
$\frac{15}{25} = \frac{3}{5}$ **Ans.**

7. The charge for 5 items is 3 + 2 + 1 + 1 + 1 = \$8. Sending 5 items twice makes the charge \$16. The charge for two items is 3 + 2 = \$5. The charge for doing this 5 times is \$25.
25 - 16 = 9 **Ans.**

8. $1\frac{1}{2} - 1\frac{1}{8} = \frac{3}{8}$
$\frac{\frac{3}{8}}{2} = \frac{3}{16}$ **Ans.**

9. Clearly 0.2 can't be greater than any of the other numbers that can be made. So look at 0.4. 0.4 > 0.26 but 0.4 < 0.62. Now look at 0.6. 0.6 is bigger than anything that can be made, i.e., 0.6 > 0.24 and 0.6 > 0.42. Thus there are 3 different ways. 3 **Ans.**

10. The line y = 3 - 2x is plotted as follows:

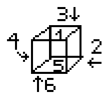


As can be seen the line goes through 3 quadrants. 3 **Ans.**

11. $\frac{2250}{360} = \frac{225}{36} = \frac{25}{4} = 6\frac{1}{4}$
The skater goes through $6\frac{1}{4}$ revolutions. If the skater started facing north and spins to her right, she'll end up facing east.**Ans.**

12. Start by placing the square with 5 on the bottom and then fold up the sides. The cube

looks like this:



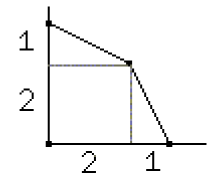
The 4 became the left side, the 3 became the back side of the cube, the 6 the front side and the 2 the right side. Finally, the 1 is on the top of the cube. The four faces adjacent to the side with the 1 are 2, 3, 4, and 6.
$2 \times 3 \times 4 \times 6 = 144$ **Ans.**

13. The Postal Service charges an extra $0.11 in postage if the length of an envelope, in inches, divided by its height, in inches, is less than 1.3 or greater than 2.5
Envelope A has a length of 6 inches and height of 4.
$\frac{6}{4} = 1.5$ which does not cost more.
Envelope B has a length of 9 inches and a height of 3 inches.
$\frac{9}{3} = 3 > 2.5$ which does cost more.
Envelope C has a length of 6 inches and a height of 6 inches.
$\frac{6}{6} = 1 < 1.3$ which does cost more.
Envelope D has a length of 11 inches and a height of 4 inches.
$\frac{11}{4} = 2.75 > 2.5$ which does cost more.
Envelopes B, C, and D cost more. 3 **Ans.**

14. A rectangle has a width of 3 units and a length of 2x + 2 units. The area is 48 square units.
$3 \times (2x + 2) = 48$
$6x + 6 = 48$
$6x = 42$
$x = 7$ **Ans.**

15. Three people share a 12.5 mile ride in the taxi cab. The first person costs $3 for the first mile and then $0.20 for each additional $\frac{1}{10}$ of a mile.
$12.5 - 1 = 11.5 = \frac{115}{10}$
Thus the charge for the first person is:
$3 + (115 \times 0.20) = 3 + 23.00 = 26$
The other people are $2 each regardless of the length of the trip.
$26 + (2 \times 2) = 26 + 4 = 30$ **Ans.**

16. The quadrilateral looks as follows:



The quadrilateral can be broken into a square with side length 2 units and 2 right triangles each with legs of 2 and 1 units. Placing the two triangles together on their hypotenuse creates a rectangle with sides of length 2 and 1.
$(2 \times 2) + (2 \times 1) = 4 + 2 = 6$ **Ans.**

17. The secret number is a factor of 30. The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30. The secret number is not a prime number. This lets out 2, 3, and 5, leaving 1, 6, 10, 15, and 30.
The secret number is not a multiple of 3. This lets out 6, 15 and 30, leaving 1 and 10. The secret number is not less than 3. This lets out 1 and we are left with 10. **Ans.**

18. $1 + 3 + 5 + ... + 31 + 33 + 35 + 37 + 39 =$
$(1 + 39) + (3 + 37) + ... + (19 + 21) =$
$40 \times 10 = 400$ **Ans.**

19. Samantha uses exactly 70 non-overlapping square tiles, each 1 cm by 1 cm, to make three squares. To determine what these squares could be, list all squares under 70: 1, 4, 9, 16, 25, 36, 49, and 64. If 64 is chosen then two other numbers must sum to 6. That can't happen.
If 49 is chosen, two other numbers must sum to 21. That can't happen.
If 36 is chosen, two other numbers must sum to 34. 9 + 25 = 34
The area of the largest square is 36. **Ans.**

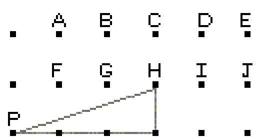
20. P and Q are each a distinct member of the set $\{6, -\frac{1}{2}, -3, \frac{1}{3}\}$. Determine the least possible value of the quotient $P \div Q$. Least possible means negative with largest absolute value. This means the signs of P and Q must differ and the absolute value of the divisor must be as small as possible.
Therefore, the divisor is $-\frac{1}{2}$. The dividend

must be the largest possible positive value or 6.

$\frac{6}{-\frac{1}{2}} = 6 \times -2 = -12$ **Ans.**

21. Three consecutive positive prime numbers have a sum that is a multiple of 7. Try listing the set of prime numbers under 100.
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
2 + 3 + 5 = 10
3 + 5 + 7 = 15
5 + 7 + 11 = 23
7 + 11 + 13 = 31
11 + 13 + 17 = 41
13 + 17 + 19 = 49 **Ans.**

22. Having $\sqrt{10}$ for a length means the side must be a hypoteneuse since the other two sides will be integral. The square of $\sqrt{10}$ is 10. Therefore, the sum of the squares of 2 integers must be 10. These can only be 9 and 1 which means the sides are 3 and 1.



Three points to the right of point P and one point up is point H. **Ans.**

23. The first 3 squares are single digits (1, 4, 9). The next 6 squares are double digits (16, 25, 36, 49, 64, 81). The rest of the squares must be triple digits (100, 121, 144, 169, 196 and 225).
$3 + (2 \times 6) + (3 \times 6) = 3 + 12 + 18 = 33$ **Ans.**

24. Two cylindrical cans have the same volume. The height of one can is triple the height of the other. The radius of the narrower can is 12 units. The formula for the volume of a cylinder is:
$V = \pi r^2 h$
Let h_1 = the height of the first cylinder.
Let h_2 = the height of the second cylinder.
Let r_1 = the radius of the first cylinder.
Let r_2 = the radius of the second cylinder.
$r_1 = 12$
$h_1 = 3h_2$
$\pi r_1^2 h_1 = \pi r_2^2 h_2$
$r_1^2 h_1 = r_2^2 h_2$
$r_1^2 \times 3h_2 = r_2^2 h_2$
$3r_1^2 = r_2^2$
$3 \times (12)^2 = r_2^2$
$r_2 = 12\sqrt{3}$ **Ans.**

25. Sara has a bag which has 10 red marbles and 12 white marbles. Each time she draws out a red marble she puts the marble back and adds five more red marbles. Each time she draws out a white marble, she puts it back and adds three more white marbles. Thus, each time the number of marbles in the bag grows. There are 10 + 12 = 22 marbles in the bag at the start and there must be 43 marbles in the bag at the finish. 43 - 22 = 21
How can 21 marbles be added in the smallest number of tries?
$5 \times 4 + 1 = 21$; No good
$5 \times 3 + 6 = 21 = (5 \times 3) + (3 \times 2) = 21$
Pick 3 red marbles and gain 15 marbles.
Pick 2 white marbles and gain 6 more marbles.
3 + 2 = 5 **Ans.**

26. Two lines intersect once. Three lines intersect 3 times. (1, 2), (1, 3) and (2, 3) (where (x,y) means line x intersects with line y). Four lines intersect 6 times. (1, 2) (1, 3), (1, 4), (2, 3), (2, 4) and (3, 4). See the pattern?

Lines:	2	3	4	5	6	7	8
Intersections:	1	3	6	10	15	21	28

28 **Ans.**

27. Let x be the ten's digit and y the one's digit. Then:
$(10x + y) \times \frac{6}{5} = 10y + x$
$(10x + y) \times 6 = 50y + 5x$
$60x + 6y = 50y + 5x$
$55x = 44y$
$x = \frac{44}{55}\ y = \frac{4}{5}y$
Since x and y must be whole numbers, x = 4 and y = 5. 45 **Ans.**

28. Let each of the smaller circles have a radius of r. Then the area of the 7 shaded circles is $7\pi r^2$. The radius of the large circle is 3r. (Create a diagonal going through the center of the circle from top to bottom and it goes through 3 circles on their diagonals.) Thus the area of the larger circle is $9\pi r^2$. The ratio of the 7 smaller circles to the larger

circle is $\frac{7}{9}$. **Ans.**

29. Team A has a $\frac{2}{3}$ probability of competing on any given day. The possibilities are that they compete on day 1 and 2, day 1 and 3, day 2 and 3 or on all 3 days. The probability that they compete on day 1/day 2 is:

$$\frac{2}{3}\times\frac{2}{3}\times\frac{1}{3}=\frac{4}{27}$$

This is the same probability of competing on day 1/day 3 or competing on day 2/day 3. The probability that they compete on all 3 days is:

$$\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}=\frac{8}{27}$$

Thus, the probability of any of these four scenarios is:

$$3\times\frac{4}{27}+\frac{8}{27}=\frac{12}{27}+\frac{8}{27}=\frac{20}{27}$$ **Ans.**

30. When all four-digit positive integers written with the digits 1, 2, 3, and 4 being used exactly once are enumerated, the values 1, 2, 3, and 4 will appear in each column exactly 6 times. (Consider 1234, 1243, 1324, 1342, 1423 and 1432. 2, 3, and 4 appear twice in the ones, tens and hundreds column. 1 appears 6 times in the thousands column. Similarly for starting the value with 2, 3, and 4. Thus, 1 appears 6 times in the ones, tens and hundreds column [when the number starts with 2, 3, or 4] and 6 times in the thousands column [when the number starts with 1]). Notice there are $4 \times 3 \times 2 \times 1 = 24$ possible four-digit numbers that can be formed, and it would follow that in ¼ (or 6) of those "1" is in the thousands place, in ¼ (of 6) of those "1" is in the hundreds place, etc. So the sum of the units column is 6(1) + 6(2) + 6(3) + 6(4) = 60, and the ones digit of the sum will be 0 and a 6 must be carried to the 10's column. For the tens column, we'll have 60 + 6 = 66. Thus, the tens digit of the sum will be 6 and 6 is carried. The hundreds digit of the sum will be 6 and 6 is carried. The thousands digit of the sum will be 6 and 6 is carried.
66,660 **Ans.**

TARGET ROUND QUESTIONS

1. The sum of two positive integers is 9. We must find the least possible sum of their reciprocals. This occurs when the two numbers are close together (because otherwise you have one reciprocal which is very small and one which is very large, at least in a relative sense -- for example consider the reciprocals of 1 and 8). The way to get both values close together is to pick 4 and 5.

$$\frac{1}{4}+\frac{1}{5}=\frac{5}{20}+\frac{4}{20}+\frac{9}{20}$$

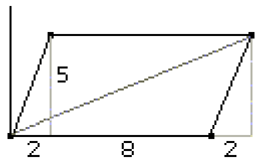
$$\frac{9}{20}=0.45$$ **Ans.**

2. Two integers are relatively prime if they have no common factors other than 1 or -1. We must find the number of integers less than or equal to 30 that are relatively prime to 30.
Any value which is a factor of 30 (other than 1) cannot be relatively prime. This removes 2, 3, 5, 6, 10 15, and 30.
Any multiple of 2 can't be relatively prime, thereby removing 4, 8, 12, 14, 16, 18, 20, 22, 24, 26, and 28 (2, 6, 10 and 30 were already counted).
Any multiple of 3 can't be relatively prime either, thereby removing 9, 21, and 27.
Any multiple of 5 can't be relatively prime as well, thereby removing 25.
This leaves 1, 7, 11, 13, 17, 19, 23 and 29.
There are 8 choices out of 30 possibilities.

$$\frac{8}{30}=\frac{4}{15}$$ **Ans.**

3. B(B - C) = 23, B and C are positive integers. 23 is a prime number. Therefore, either B is 1 and B-C is 23 or B-C is 1 and B is 23.
Start with the first scenario:
B = 1
B - C = 23
1 - C = 23
C = -22, but C is positive. Try the other scenario.
B = 23
B - C = 1
23 - C = 1
C = 22 **Ans.**

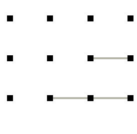
4. First graph the parallelogram.



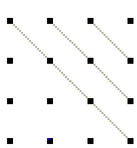
The points are (0, 0), (2 ,5), (m, n) and (10, 0). (m,n) must be (12, 5) to complete the parallelogram. The diagonal is the hypotenuse of a right triangle with sides 5 and 12. The Pythagorean Theorem tells us the diagonal 13. **<u>Ans.</u>**

5. The given sequence is:
16, 80, 48, 64, A, B, C, D
Each term beyond the second term is the arithmetic mean of the previous two terms.
Thus, 48 = (80 + 18) ÷ 2 and
64 = (80 + 48) ÷ 2
A = (48 + 64) ÷ 2 = 112 ÷ 2 = 56
B = (64 + 56) ÷ 2 = 120 ÷ 2 = 60
C = (56 + 60) ÷ 2 = 116 ÷ 2 = 58
D = (60 + 58) ÷ 2 = 118 ÷ 2 = 59 **<u>Ans.</u>**

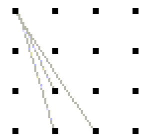
6. We are asked to find the greatest number of segments that can be drawn, using pairs of these points as endpoints, such that no two segments are the same length. The first figures shows horizontal lines going through the points. (Verticals would be no different for the size.)



There are lines of sizes 1, 2, and 3. The second figure shows the three possible diagonals through squares of sides 1, 2, and 3.



The third figure shows the three possible diagonals through rectangles of size 1x2, 1x3 and 2x3.



3 + 3 + 3 = 9 **<u>Ans.</u>**

7. Two distinct numbers are selected simultaneously and at random from the set {1, 2, 3, 4, 5}. There are only $_5C_2$ = (5!)/(2!3!) = 10 possible pairings of the numbers, since the order does not matter. We must find those pairings whose product is an even number. Only when at least one value is even can we get an even product. Thus we need to have the 2 and/or 4:
(2, 1), (2,3), (2,4), (2,5), (4,1), (4,2), (4, 3) and (4,5). Don't forget that (2,4) is the same as (4,2), so only 7 of the 10 possible pairings have an even product. $\frac{7}{10}$ **<u>Ans.</u>**

8. The first company is America Online with 64.3. If we are looking for a ratio of 2:1, then the other company must be close to 32. Try Lycos.
$$\frac{64.3}{31.8} \approx 2.02012579$$
Microsoft is at 56.2. Half of that is 28.1. Try Excite.
$$\frac{56.2}{28.8} \approx 1.951388889$$
Yahoo is at 55 million. Half of that is 27.5. Try Excite.
$$\frac{55.0}{28.8} \approx 1.909722222$$
Lycos is at 31.8. Half of that is under 16 and there is no company at that level. The first ratio of America Online to Lycos is closest to 2:1 and the smaller company is Lycos. **<u>Ans.</u>**

<u>TEAM ROUND QUESTIONS</u>

1. The measures of the four interior angles of a quadrilateral are x, 2x, x + 20 and x + 40. The total number of degrees in the angles of a quadrilateral is 360.
360 = x + 2x + x + 20 + x + 40
360 = 5x + 60
5x = 300
x = 60, which is the smallest angle. **<u>Ans.</u>**

2. A recipe that makes 48 cookies uses $2\frac{1}{4}$ cups of flour. A 5 pound bag of flour contains 20 cups. We need to make 960 cookies.

$$\frac{960}{x} = \frac{48}{2\frac{1}{4}}$$

$48x = 2\frac{1}{4} \times 960 = \frac{9}{4} \times 960$

$48x = 9 \times 240 = 2160$

$x = \frac{2160}{48} = 45$

It takes 45 cups of flour to make the cookies.

$\frac{45}{20} = 2\frac{5}{20} = 2.25$ bags of flour

Each bag is 5 pounds of flour.

$2.25 \times 5 == 11.25$ **Ans.**

3. Number the rows 1-8 and the columns 1-8. Consider, first, squares containing 4 black squares. The first square would be composed of the squares in columns 1-3 and rows 1-3 Using (row,column) as the notation., we can count from left to right: {(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)}. The next square with four black inner squares starts in column 3, then the next in column 5. So there are 3 squares using rows 1-3. (Using column 2 and looking at a square containing 9 inner squares contains 5 black squares so we're not there yet.) Now try rows 2-4. The 3x3 squares start in columns 2, 4, and 6. Continuing on, there are 3 such squares in rows 3-5, 4-6, 5-7 and 6-8, making a total of 18 squares containing 4 black inner squares. Now look at 5: Using rows 1-3: columns 2-5, 4-6, and 6-8 we can create 3x3 squares that have 5 inner black squares. Looking at rows 2-4: columns 1-3, 3-5, 5-7 do the same. So again, there will be 18 squares, this time each containing 5 inner black squares.
4x4 squares always contain 8 black squares. There are 5 of these in rows 1-4, and similarly 5 in each of 2-5, 3-6, 4-7 and 5-8 for a total of 25 squares.
5x5 squares contain either 12 or 13 inner black squares. There are 4 of these in rows 1-5 and similarly, 3 in each of rows 2-6, 3-7 and 4-8 for a total of 16 squares.
6x6 squares contain 18 black squares. There are 3 of these in rows 1-6, 2-7 and 3-8 for a total of 9. (see the squares pattern?).
Finally there are 4 squares size 7x7 and 1 of size 8x8.
36 + 25 + 16 + 9 + 4 + 1 = 91 **Ans.**

4. Using the riding mower, the yard can be mowed in 45 min. or $\frac{1}{45}$ of the yard each minute. Using the push mower, the yard can be mowed in 105 min. or $\frac{1}{105}$ of the yard per minute. If Monika and Marcelo mow together than

$\frac{1}{45} + \frac{1}{105} = \frac{7}{315} + \frac{3}{315} = \frac{10}{315} = \frac{2}{63}$ of the yard is done per minute. (315 is the LCM of 45 and 105.)

Let x = the number of minutes it takes to mow the yard using both mowers.

$\frac{2}{63}x = 1$

$x = \frac{63}{2} = 31.5$ minutes. **Ans.**

5. How many positive, even three-digit numbers exist such that the sum of the hundreds digit and the tens digit equals the units digit?
Start by looking at the numbers between 100 and 199.
The units digit can never be 0.
112, 134, 156, and 178 are the only numbers that satisfy the requirement. That's 4 numbers.
Now look at the numbers between 200 and 299. In this case the units digit can never be 0 or 1.
202, 224, 246, 268, and there are 4 numbers.
For 300 - 399, there are 314, 336, and 357 or 3 numbers. For 400-499, there are 404, 426 and 448 or 3 more. See the pattern?
For 800 - 899, the only number is 808 and for 900 - 999, there are none (since 909 is odd).
4 + 4 + 3 + 3 + 2 + 2 + 1 + 1 = 20 **Ans.**

6. A 25 square foot region has $25 \times 12 \times 12$ square inches. Therefore, each plant has:

$\frac{25\times12\times12}{10\times12} = \frac{25\times12}{10} = \frac{5\times12}{2} =$

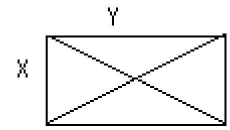
$5 \times 6 = 30$ sq. in. per plant.

30 **Ans.**

7. How many zeroes are at the end of 100!?
$100! = 100 \times 99 \times 98 \times ... \times 3 \times 2 \times 1$
To see how to solve this, realize that factoring each of the numbers and looking for sets of 2 and 5 (to get 10) will determine the value.
How many of these values have 5 as factors? Clearly, all multiples of 5 and there are 20 of those. But wait a minute. Some of them, like 25, 50, 75 and 100 have two factors of 5 since $25 = 5 \times 5$. So there are 24 5's that can be factored out of 100!.
How many 2's are there? Certainly a minimum of 50 (for the 50 even numbers) but there are definitely more. In this case, it really doesn't matter since the limitation will be the number of 5's that are factors.
Therefore 100! will end in 24 0's.
How about 200!? There are already 24 from 100 on down so just consider the factors 200 down to 101. As before there are 20 numbers that are multiples of 5 and 4 more that are multiples of 25. So it must be 24 again? No, not quite. $125 = 5 \times 5 \times 5$
There's an extra one, or 25. Therefore 200! has 24 + 25 = 49 5's and will end in 49 0's.
Now, look at 300!. We already know that there are 49 from 200 down to 1 so just consider 300 to 201. Obviously, there are 20 more multiples of 5 and 4 more that are multiples of 25. Is it just 24? No. $250 = 125 \times 2$ so there is one more 5 for a total of 25. So, 300! has 25 + 49 = 74 factors of 5 and ends in 74 zeroes.
We are multiplying numbers that end in 24 zeroes, 49 zeroes and 74 zeroes, respectively. This means that the final product ends in 24 + 49 + 74 = 147 zeroes.
147 **<u>Ans.</u>**

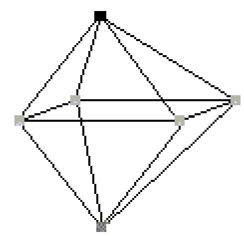
8. To find the median positive difference, calculate all 5 positive differences.
Jacksonville: 122.35 - 93.65 = 28.7
Miami: 144.24 - 129.20 = 15.04
Orlando: 112.25 - 91.98 = 20.27
Tallahassee: 161.18 - 110.81 = 50.37
Tampa: 102.29 - 76.10 = 26.19
The median is the middle value which is 26.19. **<u>Ans.</u>**

9. Let x and y be the length and width of a rectangle.
$2(x + y) = 34$
$x + y = 17$
$xy = 60$



Drawing a diagonal of the rectangle gives two right triangles with sides x and y and hypotenuse of $\sqrt{x^2 + y^2}$
The hypotenuse of the right triangle is the diagonal and the product of both is $x^2 + y^2$.
$(x + y)^2 = x^2 + y^2 + 2xy$
$17^2 = x^2 + y^2 + (2 \times 60)$
$289 = x^2 + y^2 + 120$
$x^2 + y^2 = 169$ **<u>Ans.</u>**

10. The ant starts at the top vertex (black in the figure) and walks to an adjacent vertex (light gray in the figure).



Any one of the light gray vertices is a possible vertex A. From vertex A, he walks to one of the 4 available vertices. What's the possibility that this vertex is the bottom vertice? From any of the light gray vertices, the ant has 2 choices that are light gray, one that is black and one that is dark gray at the bottom. So he has a $\frac{1}{4}$ probability that he lands on the dark gray vertex at the bottom.
$\frac{1}{4}$ **<u>Ans.</u>**