2003 STATE COMPETITION

SPRINT ROUND QUESTIONS

1. Row A shows the integers 4 and 10. Row B shows the integers 3, 5, 9, 11. Row C shows the integers 2, 6, 8, 12, 14.
   Row D shows the integers 1, 7, 13. 14 will appear in Row C, 15 in Row B and 16 in Row A. So Row A now as 4, 10 and 16. The pattern emerges. Each integer in Row A is 6 greater than the previous integer in Row A. And the first integer is 2 less than 6. So the value can be expressed as 6x - 2. Therefore, the eighth value in Row A is:
   $6 \times 8 - 2 = 48 - 2 = 46$. **Ans.**

2. First, assume there are no leap years. Later, we will see whether it matters or not. There are 365 days in a year. $\frac{365 \times 7}{7} = 365$ with no remainder. It does not matter whether there is a leap year or not within the 7 years (or even two or none at all) because the additional days would not create an additional week. 365 **Ans.**

3. We are asked to find the perfect-square integer that is closest to 273. $10^2 = 100$ and $20^2 = 400$. Obviously, the value is in between. $15^2 = 225$, $16^2 = 256$ and $17^2 = 289$. It must be either 16 or 17.
   273 - 256 = 17
   289 - 273 = 16
   289 is closer by 1. 289 **Ans.**

4. There are 300 students that took the test. To find a value that is better than 75% of the scores, we must find the 225th value since 225 students are 75% of the total. That is the 76th score if we start from the highest.
   $7 + 16 = 23$ with a score ≥ 11.
   $23 + 37 = 60$ with a score ≥ 10.
   $60 + 45 = 105$ with a score ≥ 9.
   The 225th student must have scored a 9. To have a score that is higher than that, add 1. 10 **Ans.**

5. 17, a, b, c, 41 is an arithmetic sequence. This means that the difference between two values in the sequence is always the same.
   41 - 17 = 24
   This increment of 24 is reached in 4 stages so the difference between values is $\frac{1}{4} \times 24 = 6$.
   17 + 6 = 23
   23 + 6 = 29
   Therefore, b = 29. **Ans.**

6. Two complementary angles are in the ratio of 7 to 23. The complement of angle A is angle B and the complement of angle B is angle A. Thus, the ratio of angle B to angle A is just the opposite of the ratio of angle A to angle B or $\frac{23}{7}$. **Ans.**

7. In 7 years, Rich's money doubled from \$100 to \$200. In 7 more years his money will double to \$400. 7 more years will bring it to \$800 and 7 more years will bring it to \$1600.
   7 + 7 + 7 = 21 **Ans.**

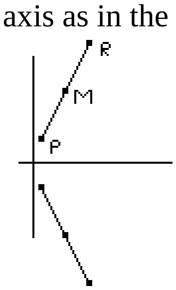
8. Find the slope of a line parallel to: 2x + 4y = -17.
   First represent the line in the form:
   y = mx + b
   4y = -2x - 17
   $y = \frac{-2x}{4} - \frac{17}{4}$
   $y = -\frac{1}{2}x - \frac{17}{4}$

The slope, or m, is $-\frac{1}{2}$. The slope of a line parallel to this line will have this exact slope.

$-\frac{1}{2}$ **Ans.**

9. How many distinct positive integers can be represented as the difference of two numbers in the set {1, 3, 5, 7, 9, 11, 13}?
Clearly, each integer subtracted from the integer directly on its right gives a difference of 2. Each integer subtracted from the integer, two values to the right, gives a difference of 4. Similarly, we can get differences of 6, 8, 10 and 12 for a total of 6 integers. 6 **Ans.**

10. Point P is located at (1,3) and Point R is located at (7,15). Point M is the midpoint of segment PR. The x-coordinate of the M is:
$7 - 1 = 6$
$6 \div 2 = 3$
$1 + 3 = 4$
The y-coordinate of M is:
$15 - 3 = 12$
$12 \div 2 = 6$
$3 + 6 = 9$
So M is (4, 9).
Segment PR is reflected over the x-axis as in the figure below.

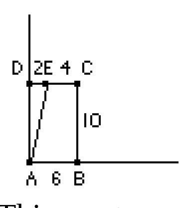


Each point has the same x and y coordinates **except** the y-coordinate is negated. So the reflected point for M is:
(4, -9).
$4 + -9 = -5$ **Ans.**

11. In a three-digit number, the hundreds digit is greater than 5, the tens digit is greater than 4 but less than 8 and the units digit is the smallest prime number. How many three-digit numbers satisfy all of these conditions?
The hundreds digit is greater than 5 which means 6, 7, 8, or 9 for a total of 4 possible values. The tens digit is greater than 4 but less than 8 or 5, 6, 7 for a total of 3 different values. The units digit is the smallest prime number or 2. This is a total of only one digit.
$4 \times 3 \times 1 = 12$ **Ans.**

12. The trip from Carville to Nikpath requires $4\frac{1}{2}$ hours when traveling at an average speed of 70 miles per hour. This means the total distance is $4\frac{1}{2} \times 70 = \frac{9}{2} \times 70 = 9 \times 35 = 315$ miles. If traveling at a speed of 60 miles per hour, the trip must take:
$\frac{315}{60} = 5\frac{15}{60} = 5\frac{1}{4} = 5.25$ **Ans.**

13. Rectangle ABCD has the vertices A(0,0), B(6,0), C(6,10), and D(0,10). The point E is on segment CD at (2,10). A line connecting E with A is drawn as in the figure below.



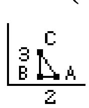
This creates a right triangle, ADE, with two of the sides being 10 and 2. The area of the triangle is:
$\frac{1}{2} \times 10 \times 2 = 10$

The area of quadrilateral ABCE is the area of triangle ADE subtracted from the area of rectangle ABCD.
The area of ABCD is $10 \times 6 = 60$.
The area of quadrilateral ABCE is:
60 - 10 = 50
The ratio of the area of triangle ADE to the area of quadrilateral ABCE is:
$\frac{10}{50} = \frac{1}{5}$ **Ans.**

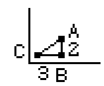
14. The digits 2, 4, 6 and 9 are placed such that two are in the numerator and two are in the denominator. We are asked to find the smallest of all the common fractions that can be formed. The smallest value will occur when the smallest value is on top and the largest value is on the bottom. Choose 24 for the numerator and 96 for the denominator.
$\frac{24}{96} = \frac{1}{4}$ **Ans.**

15. At 10 AM, Boon Tee is the 225th person in line to ride the roller coatser. Each roller coaster train holds 36 people and a full train leaves every four minutes. The first 36 people in line leave on the10:01 train. First determine which group Boon Tee will be in.
$\frac{225}{36}$ = 6 R 9.
Boon Tee will be in the 7th group. The first group leaves at 10:01. It will take $6 \times 4 = 24$ more minutes until Boon Tee can leave.
10:01 + 24 = 10:25 **Ans.**

16. A triangle has vertices A(6,1), B(4,1) and C(4,4) as shown in the picture.



The triangle is a right triangle with horizontal length, 2, and vertical length, 3. The triangle is rotated 90 degrees counterclockwise around B. This means the triangle will have vertical length, 2, and horizontal length 3, as in this picture.



Since B is at (4,1), A must be 2 units higher, or (4,3). C is now 3 units to the left of B or (1,1). **Ans.**

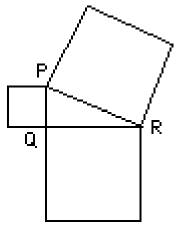
17. What is the least natural number that can be added to 40,305 to create a palindrome? A palindrome is the same number when read forwards or backwards. The nearest palindrome would be 40,304 but we must find the next palindrome greater than 40,305. There can be no more palindromes in the 40,300 range. So we must look at 40,400 and larger. The next palindrome is 40,404.
40404 - 40305 = 99 **Ans.**

18. A @ B = $\frac{A}{B} + A \times B$
4 @ 2 = $\frac{4}{2} + 4 \times 2 = 2 + 8 = 10$
20 @ 10 = $\frac{20}{10} + 20 \times 10$
= 2 + 200 = 202 **Ans.**

19. Numbers on a standard six-faced die are arranged such that numbers on opposite faces always add to seven. We need to find the largest product of all faces except the top and bottom. Clearly the larger the number the larger the product will be. But we are limited by opposing values having to add to 7. Certainly the ideal is to get rid of a 1 in the product. So we remove 6 and 1.

$5 \times 4 \times 3 \times 2 = 120$ **<u>Ans.</u>**

20. Angle PQR is a right angle. The three quadrilaterals shown are squares.



The sum of the areas of the three squares is 338 square centimeters. We must find the area of the largest square. What we are looking for, then, is the addition of three perfect squares to equal 338. In addition triangle PQR is a right triangle so these values are probably one of our favorite right triangles which are:
3, 4, 5
6, 8, 10
5, 12, 13
8, 15, 17
The square of 5 is 25. This will be too small.
The square of 10 is 100. The sum will be too small.
The square of 13 is 169, 12 is 144 and 5 is 25.
169 + 144 + 25 = 338 Bingo!
169 **<u>Ans.</u>**

21. Container I holds 8 red balls and 4 green balls.
Container II holds 2 red balls and 4 green balls.
Container III also holds 2 red balls and 4 green balls.
We selected a container and a ball at random and must find the probability that the selected ball is green.
The probability of Container I being selected is $\frac{1}{3}$. The probability of picking a green ball from Container I is $\frac{4}{8+4} = \frac{4}{12} = \frac{1}{3}$ Thus, the probability of selecting a green ball from Container I is $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.
The probability of selecting Container II is $\frac{1}{3}$. The probability of selecting a green ball is $\frac{4}{2+4} = \frac{4}{6} = \frac{2}{3}$.
Thus, the probability of selecting a green ball from Container II is $\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$.
The probability of selecting a green ball from Container III is the same as the probability of selecting a green ball from Container II or $\frac{2}{9}$.
$\frac{1}{9} + \frac{2}{9} + \frac{2}{9} = \frac{5}{9}$ **<u>Ans.</u>**

22. The arithmetic mean of nine numbers is 54. If two numbers u and v are added to the list, the mean of the eleven-member list becomes 66. Since the mean of the original nine numbers is 54, the sum of all 9 values must be $54 \times 9 = 486$.
Adding two numbers gives a mean of 66, so the sum of all 11 numbers is 726.
726 - 486 = 240
240 is the sum of u and v. Thus, the mean of u and v is $\frac{240}{2}$ = 120. **<u>Ans.</u>**

23. A gasoline gauge originally read $\frac{1}{8}$ full. Then, 15 gallons of gasoline were added and the gauge read $\frac{3}{4}$ full. Thus,
$\frac{3}{4} - \frac{1}{8} = \frac{6}{8} - \frac{1}{8} = \frac{5}{8}$ of the tank holds 15

gallons of gas. So $\frac{1}{8}$ of the tank holds $\frac{15}{5}$ = 3 gallons. Since the tank is $\frac{3}{4}$ full it is actually $\frac{1}{4} = \frac{2}{8}$ empty.
$2 \times 3 = 6$ **Ans.**

24. Three fair, standard six-faced dice of different colors are rolled. In how many ways can the dice be rolled such that the sum of the numbers rolled is 10? Below is a table. No combination will be repeated.

1	**2**	**3**
1	1	8 Impossible!
1	2	7 Impossible!
1	3	6 6 ways
1	4	5 6 ways
2	2	6 3 ways
2	3	5 6 ways
2	4	4 3 ways
3	3	4 3 ways

Any other combination has already been entered.
6 + 6 + 3 + 6 + 3 + 3 = 27 **Ans.**

25. Arlene skips all multiples of 3 and all numbers that contain the digit 3. In the first 10 numbers there is one number containing 3 and 3 multiples of 3 (one of which is 3) so 3 numbers are discounted. Therefore there are 7 numbers between 1 and 10.
In the next 10 numbers, i.e., 11-20, there are 3 multiples divisible by 3 and one number containing 3 (that is not divisible by 3), so 4 numbers are discounted, leaving 6. 7 + 6 = 13.
In the next 10 numbers, i.e., 21-30 there are 4 numbers divisible by 3 and one number containing 3 (that is not divisible by 3, i.e., 23) and one number containing 3 (that is divisible by 3 and so is already counted, i.e., 30) so 5 numbers are discounted. 13 + 5 = 18
In the next 10 numbers, i.e., 31-40 only one number has no 3 in it and it (40) is not divisible by 3 so only 1 number is discounted. 18 + 1 = 19
Starting with 41-50 the pattern emerges - 7, 6, 5 for every thirty numbers.
19 + 6 = 25 (41-50)
25 + 5 = 30 (51-60)
30 + 7 = 37 (61-70)
Since we are looking for the 40th number, there are only 3 to go. 71 is the 38th number. 72 is divisible by 3 and 73 ends in 3. Therefore, the 39th number is 74. 75 is divisible by 3 so the 40th number is 76. **Ans.**

26. n^2 gives a remainder of 4 when divided by 5 and n^3 gives a remainder of 2 when divided by 5.
If n^2 gives a remainder of 4 when divided by 5, the number must have a square which ends in either 4 or 9. Then, if n^3 gives a remainder of 2 when divided by 5, the number must also have a cube which ends in either 2 or 7.
Case I: Numbers that, when squared, end in 4, must end in 2 or 8. $2^3 = 8$ so a number ending in 2, when cubed would end in 8. So n cannot end in 2. $8^3 = 512$ so the number could end in 8.
Case II: Numbers that, when squared, end in 9, must end in 3 or 7. $7^3 = 343$ so a number ending in 7, when cubed would end in 3. So n cannot end in 7. $3^3 = 27$ so the number could end in 3.
When any number ending in 8 or 3 is divided by 5, the remainder will be 3. **Ans.**

27. $\frac{x}{y} = \frac{4}{7}$

$\frac{y}{z} = \frac{14}{3}$

$\frac{x}{z} = \frac{x}{y} \times \frac{y}{z} = \frac{4}{7} \times \frac{14}{3} = \frac{4 \times 2}{3} = \frac{8}{3}$

$\frac{x}{z} + \frac{y}{z} = \frac{x+y}{z} = \frac{8}{3} + \frac{14}{3} = \frac{22}{3}$ **Ans.**

28. Two different numbers are randomly selected from the set
S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}
The probability that their sum is 12 would be greater if the number *n* had first been removed from the set.
If we remove 1, the pairs whose sum would result in 12 are (2,10), (3, 9), (4, 8), (5, 7) or 4 choices.
If 2 is removed from the set, the choices whose sum would result in 12 are (1, 11), (3, 9), (4, 8), (5, 7) or 4 choices.
If 3 is removed from the set, the pairs are (1, 11), (2, 10), (4, 8), (5, 7) or 4 choices. Similarly for 4 and 5. What about 6?
The choices are (1, 11), (2, 10), (3, 9), (4, 8), (5, 7) for 5 pairs. (This is because 6 + 6 = 12 and we can't choose the same value twice.)
Similarly, removing 7, 8, 9, 10, or 11 would give us 8 choices each time.
6 **Ans.**

29. The clock reads 4:20 and the second hand makes one complete circle every 4 seconds. The minute hand and hour hand behave normally within this context. Tomas came into the room at 9:00 a.m. and will leave at 9:50 a.m.
50 minutes = 50 × 60 = 3000 seconds.
Each 4 seconds are reflected by 60 seconds of movement in the clock. This means that the clock moves
$\frac{3000}{4} \times 60$ = 45000 seconds or
$\frac{45000}{60} = 750$ minutes
750 minutes is 12 hours and 30 minutes. Since the clock was at 4:20 at 9, the clock will be at 4:50 when Tomas leaves. 4:50 **Ans.**

30. $x + \frac{1}{x} = 4$

$\left(x + \frac{1}{x}\right)^2 = x^2 + 2 + \frac{1}{x^2} = 16$

$\left(x + \frac{1}{x}\right)^3 = \left(x^2 + 2 + \frac{1}{x^2}\right)\left(x + \frac{1}{x}\right) = 64 =$

$x^3 + 2x + \frac{x}{x^2} + \frac{x^2}{x} + \frac{2}{x} + \frac{1}{x^3} =$

$x^3 + 2x + \frac{1}{x} + x + \frac{2}{x} + \frac{1}{x^3} =$

$\left(x^3 + \frac{1}{x^3}\right) + \left(x + \frac{1}{x}\right) + 2\left(x + \frac{1}{x}\right) =$

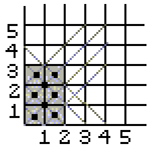
$\left(x^3 + \frac{1}{x^3}\right) + 3\left(x + \frac{1}{x}\right) = 64$

$\left(x^3 + \frac{1}{x^3}\right)$ = 64 - (3 × 4) = 64 -12 =
52 **Ans.**

TARGET ROUND QUESTIONS

1. A stack of 100 nickels is 6.25 inches high.
$\frac{100}{6.25} = \frac{x}{12 \times 8} = \frac{x}{96}$
6.25x = 96 × 100 = 9600
x = $\frac{9600}{6.25}$ = 1536 nickels
$\frac{1536}{20}$ = $76.80 **Ans.**

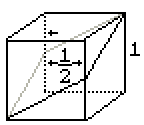
2. In the picture below, all lines with integer y-intercepts and slope 1 or -1 are drawn.



These lines are y = x, y = x + 1, y = x + 2, etc. and y = -x + 2, y = -x + 3, etc. As is evident, there are 8 intersection points within the shaded region.
8 **Ans.**

3. The sum of 9 consecutive integers is 9. Let x be the fifth, or middle one.
x - 4 + x - 3 + x - 2 + x - 1 + x + x + 1 + x + 2 + x + 3 + x + 4 = 9
9x = 9
x = 1
x - 4 = 1 - 4 = -3 **Ans.**

4. A cube is sliced by a plane which goes through two opposite corners and the midpoint of two edges as shown.



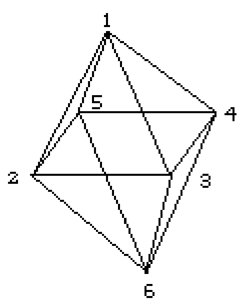
The edge of the cube has length one unit. So each side of the rhombus is the hypotenuse of a triangle whose other sides are 1 and $\frac{1}{2}$.
The area of a rhombus is $\frac{1}{2}d_1d_2$ where d_1 and d_2 are the diagonals of the rhombus. The shorter diagonal, d_1, is just the equivalent of a diagonal of a face of the cube.
$d_1^2 = 1^2 + 1^2 = 1 + 1 = 2$
$d_1 = \sqrt{2}$
The longer diagonal, d_2, is the diagonal from the bottom of the cube on one side to the top of the cube on the other side. This diagonal is the hypotenuse of a right triangle whose sides are an edge of the cube and a diagonal of the face (which is the same as d_1.
$d_2^2 = 1^2 + \sqrt{2}^2 = 1 + 2 = 3$
$d_2 = \sqrt{3}$
$\frac{1}{2}d_1d_2 = \frac{1}{2} \times \sqrt{2} \times \sqrt{3} = \frac{\sqrt{6}}{2}$ **Ans.**

5. 310 million of the 6.25 billion people live in North America.
$\frac{310{,}000{,}000}{6{,}250{,}000{,}000} = 0.0496$
$0.0496 \times 100 = 4.96 \approx 5$ **Ans.**

6. Paco uses a spinner to select a number from 1 through 5, each with equal probability. Manu uses a different spinner to select a number from 1 through 10, each with equal probability. We must find the number of selections whose product is less than 30. Clearly, there are 5 × 10 = 50 combinations.
If Paco chooses a 1, then Manu can choose anything for 10 combinations.
If Paco chooses a 2, then Manu can choose anything for 10 more combinations.
If Paco chooses a 3, then Manu can choose 1 through 9 for 9 more combinations.
If Paco chooses a 4, then Manu can choose 1 through 7 for 7 more combinations.
If Paco chooses a 5, then Manu can choose 1 through 5 for 5 more combinations.
10 + 10 + 9 + 7 + 5 = 41
$\frac{41}{50}$ **Ans.**

7. What is the smallest positive integer N such that the value 7 + 30 × N is not a prime number.
7 + 30 × 1 = 7 + 30 = 37 is prime
7 + 30 × 2 = 7 + 60 = 67 is prime
7 + 30 × 3 = 7 + 90 = 97 is prime
7 + 30 × 4 = 7 + 120 = 127 is prime
7 + 30 × 5 = 7 + 150 = 157 is prime
7 + 30 × 6 = 7 + 180 = 187 = 11 × 17
187 is not prime.
6 **Ans.**

8. An ant starts at the top vertex of a regular octahedron and walks along the edges of the triangles without ever traversing the same edge twice. The ant could actually traverse all edges except for the fact that the ant must stop when it returns to the starting vertex. The picture below numbers the vertices.

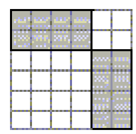


The ant can only traverse one vertex from 1 down to any of 2, 3, 4, or 5. The ant will be able to move around from 2 to 3 to 4 to 5 for a total of 4 edges and down to the bottom vertex, 6, as many times as it takes for 4 more edges. 1 + 4 + 4 = 9
One example is: 1, 2, 6, 3, 2, 5, 6, 4, 3, 1.
9 **Ans.**

TEAM ROUND QUESTIONS

1. $\frac{987670}{128}$ = 7716.17185
7716 × 128 = 987648
987670 - 987648 = 22 **Ans.**

2. We are asked to find all rectangles of area 8 square units that can be formed using only the line segments of the grid as the sides of the rectangles. This means using integral factors of 8. These are:
1 x 8 and
2 x 4
1 x 8 (or 8 x 1) is impossible because the grid is 6 x 6. 2 x 4 (or 4 x 2) is certainly possible. The picture below shows the horizontal and vertical types of rectangles that are possible.



The 2 x 4 is featured at the top left and the 4 x 2 is featured at the bottom right. First, let's determine how many 2 x 4 rectangles we can get. If you move the 2 x 4 rectangle one cube to the right you get another legal 2 x 4 rectangle. Do it once more and the rectangle will abut the edge of the cube. So, there are 3 rectangles on that line. Similarly, move the rectangle one row down. You can do that for a total of 5 times so you can have 5 × 3 = 15 of the 2 x 4 rectangles. Similarly, take the 4 x 2 rectangle and move it to the left one cube and you get another valid 4 x 2 rectangle. You can continue moving left for a total of 5 rectangles. You can also move up towards the top of the cube for a total of 3 rectangles. Again, 5 × 3 = 15.
15 + 15 = 30 **Ans.**

3. If t is an odd positive integer, then the value of the following term is:
3t - 9

If the value of a given term is an even positive integer, then the value of the following term is:
2t - 7
The terms of the sequence alternate between two positive integers (a, b, a, b, ...).
Suppose a is odd. Then the next term is 3a - 9 = b. 3a is always odd and if you subtract an odd from an odd, you get an even value so b is even. Therefore the next value must be 2t - 7 or 2(3a - 9) - 7 = 6a - 18 - 7
6a - 25 = a
5a = 25
a = 5
b = 6
This works.
Could a have been even? Then b would be 2a - 7 which would be an odd number. This would make the next term 3(2a - 7) - 9 = 6a - 21 - 9
6a - 30 = a
5a = 30
a = 6
Then, as we already know, b would have to be 5.
5 + 6 = 11 **Ans.**

4. A, B, C, D, and E are all different digits in the following addition:

	C	A	E
+		C	D
	A	B	B

Either, E + D = B or
E + D = B + 10
So E + D - B = 0 or
E + D - B = B + 10 - B = 10
10 + 0 = 10 **Ans.**

5. 3x + y = 17
5y + z = 14
3x + 5z = 41
6x + 6y + 6z = 72
6(x + y + z) = 72
x + y + z = 12 **Ans.**

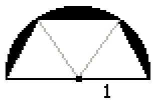
6. We have 6 different Spiderman comics, 5 different Archie comics, and 4 different Garfield comics. When stacked, each kind of comic is grouped together with others of the same kind. This means there are:
6! different combinations for the Spiderman comics,
5! different combinations for the Archie comics, and
4! different combinations for the Garfield comics.
$6! \times 5! \times 4! =$
$720 \times 120 \times 24 =$
2,073,600 Are we done? Well, not quite. There are 6 different ways that you can place the sets of comic books so:
$6 \times 2{,}073{,}600 = 12{,}441{,}600$ **Ans.**

7. P(n) is the probability than an "n" is rolled on a die.
P(1) = P(2)
P(3) = P(4) = P(5)
P(4) = 3(P(2))
P(5) = 2(P(6))
What is P(6)?

P(4) = 3(P(2))
P(3) = 3(P(2))
P(5) = 3(P(2))
P(6) = $\frac{P(5)}{2} = \frac{3P(2)}{2}$
P(1) = P(2)
P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1
$P(2) + P(2) + 3(P(2)) + 3(P(2)) + 3(P(2)) + \frac{3P(2))}{2} = 1$
$11(P(2)) + \frac{3(P(2))}{2} = 1$
$\frac{25(P(2))}{2} = 1$
P(2) = $\frac{2}{25}$

P(6) = $\frac{3(P(2))}{2} = \frac{3}{2} \times \frac{2}{25} = \frac{3}{25}$ **Ans.**

8. An isosceles trapezoid is inscribed in a semicircle as shown below.



The three shaded regions are congruent. The radius of the circle is 1. Since the trapezoid is an isosceles trapezoid and the three shaded regions are congruent, we can break the trapezoid up into 3 congruent equilateral triangles, each of side 1. Therefore, the height of each triangle is:

$h^2 + \frac{1}{2}^2 = 1^2 = 1$

$h^2 = 1 - \frac{1}{4} = \frac{3}{4}$

h = $\frac{\sqrt{3}}{2}$

The area of the triangle is

$\frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$

There are three triangles so the total area is:

$\frac{3\sqrt{3}}{4} = 1.299038106 \approx 1.3$ **Ans.**

9. What is the greatest positive integer n such that 3^n is a factor of 200!? To answer this we need to know how many multiples of 3 exist in 200!.

$\frac{200}{3}$ = 66 R 2 so there are at least 66 3's that come from factoring 200! But now we have to consider how many numbers are multiples of 9 which has two 3's in it.

$\frac{200}{9}$ = 22 R 2 so there are an additional 22 multiples of 3. Now consider how many numbers are multiples of 27 which has three 3's in it.

$\frac{200}{27}$ = 7 R 11 so there are an additional 7 multiples of 3. Next, consider how many numbers are multiples of 81 which has 4 three's in it.

$\frac{200}{81}$ = 2 R 38 so we have 2 more multiples of 3. The next power of 3 is 243 which is too large.

n = 66 + 22 + 7 + 2 = 97 **Ans.**

10. An abundant number is a positive integer, the sum of whose distinct proper factors is greater than the number. The proper factors are all the factors except the number itself. How many numbers less than 25 are abundant numbers?
Remove 1 since it has no factors other than 1. Remove all primes since they have no factors other than 1 and themselves. This leaves just the even numbers except for 2.
Factors of 4 are 1 and 2. No.
Factors of 6 are 1 2 and 3. No.
Factors of 8 are 1, 2, and 4. No.
Factors of 10 are 1, 2, and 5. No.
(Quite clearly we have to have some of the favorite numbers which have many factors but we'll keep going.)
Factors of 12 are 1, 2, 3, 4, 6. Yes!!!
Factors of 14 are 1, 2, and 7. No.
Factors of 16 are 1, 2, 4, and 8
(Perfect squares aren't good candidates, are they?)
Factors of 18 are 1, 2, 3, 6, and 9. Yes!!!
Factors of 20 are 1, 2, 4, 5, and 10. Yes!!!
Factors of 22 are 1, 2, and 11. No.
Factors of 24 are 1, 2, 3, 4, 6, 8, and 12. Definitely yes!!!

So we have 12, 18, 20, and 24.
4 **Ans.**